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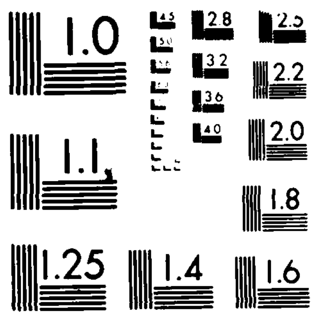

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The combination of a bumpy torus field and a conventional betatron field leads to an interesting strongly-focused, high-current accelerator configuration. The question of orbital stability of a test particle in such a device is discussed and it is shown that the alternating gradient focusing in this accelerator can easily lead to greater than 20% bandwidth in allowed mismatch between the vertical magnetic field and the average beam particle energy.

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A BUMPY TORUS BETATRON

I. INTRODUCTION

Conventional betatrons[1] are current-limited due to the defocusing effects of space charge at injection. In recent years there have been several renewed attempts at overcoming this (rather severe) space charge limit. Specifically, there have been high current conventional betatrons proposed[2] which employ high-energy injectors as well as so called modified betatrons[3,4] which employ a toroidal magnetic field to prevent space charge blow-up of the beam. In both of these cases however, a mismatch between the injection energy and vertical field of a few percent or so will cause the beam to hit the wall, a matter of some concern in a high current device. The maximum allowed error in the vertical field is typically on the order of a few gauss in designs which have been considered. Recently it was shown[5] that the combination of an $l = 2$ stellarator field and ordinary weak focusing betatron field results in a strong focusing high-current betatron or, "stellatron," with a large energy bandwidth. Such a configuration has the advantages of relaxing the vertical field and injector tolerances. In addition, the strong focusing introduces a threshold for the negative mass instability, so that this instability does not operate at injection (though other fast growing resistive or kink modes may occur below the negative mass threshold). In this note we report analytical and numerical results on the bandwidth and stability of an alternative strong-focusing scheme, namely, a combination "bumpy torus" and betatron field, corresponding to the $l = 0$ stell‥on.

II. DISCUSSION

The bumpy-torus betatron field consists of a superposition of an $l = 0$ stellarator field and the field of a conventional betatron. Near the minor axis at $r = r_0$, $z = 0$, this field has the form

$$
\begin{aligned}
B_r &= -\, nyB_{z0} + \frac{1}{2}\,\delta B_\theta\, mx \sin m\theta \\
B_\theta &= B_{\theta o}\,(1 + \frac{\delta B_\theta}{B_{\theta o}} \cos m\theta) \\
B_z &= B_{z0}(1 - nx) + \frac{1}{2}\,\delta B_\theta\, \mathrm{my} \sin m\theta
\end{aligned}
\tag{1}
$$

where $x \equiv (r - r_o)/r_o$, $y \equiv z/r_o$, θ is the azimuthal angle, n is the vertical field index, and m is the number of bumpy-torus field periods around the torus. B_{z0}, $B_{\theta o}$, and δB_θ are constants.

Treating the self fields of the beam by a simple cylindrical model, we find the equation of motion for a test particle within the beam is, in the paraxial approximation, for $n = 1/2$,

$$
\frac{d^2\psi}{d\theta_m^2} + \frac{1}{m^2}\,[2 - 4n_s + b^2(1 + \epsilon \cos 2\,\theta_m)^2]\,\psi = \frac{4}{m^2}\,\frac{\delta P}{P_o}\,e^{\frac{ib}{2m}(2\theta_m + \epsilon \sin 2\theta_m)},
\tag{2}
$$

where $\theta_m \equiv m\theta/2$, $\psi \equiv (x + iy) \exp\,[(ib/2m)\,(2\theta_m + \epsilon \sin 2\,\theta_m)]$, $b \equiv B_{\theta o}/B_{z0}$, $\epsilon \equiv \delta B_\theta/B_{\theta o}$, P_o is the momentum of a particle which would circulate on the minor axis, δP is the "momentum error," $n_s \equiv \omega_b^2/(2\gamma_o^2\Omega_{z0}^2)$ where ω_b, Ω_{z0} are the beam plasma frequency and the vertical field cyclotron frequency, respectively, and $\gamma_o = (1 + (P_o/mc)^2)^{1/2}$. We are interested both in the solution to the homogeneous part of Eq. (2), which will give orbital stability criteria, as well as in the solution to the inhomogeneous problem, which will give the momentum compaction of the machine.

The quantity n_s appearing in Eq. (2) describes the (net defocusing) effect of the self electric and magnetic forces of the beam. Since it depends on beam density and therefore on the beam minor radius, n_s will in general vary with azimuthal angle θ around the device in a manner governed by the standard beam envelop equation. Consequently, when the beam envelop is stable, we expect n_s to

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behave as $n_s(\theta) \approx n_{so} + n_{s1} \cos m\theta + \ldots$ but we shall assume here, for simplicity, that $2\epsilon b^2 >> n_{s1}$ so that, in Eq. (2), n_s may be adequately approximated by its average value.

Equation (2) is a Hill equation, which has characteristic bands of stability, as shown in Fig. 1. The boundaries of the stable regions have been obtained numerically, using standard methods.[6] The shaded regions in the figure are unstable portions of the plane, ϵ vs b/m, for the case $n_s = 30$ and $m = 30$. The intersections of the unstable regions with the abscissa are given by

$$(b^2 + 2 - 4n_s)/m^2 = q^2 \text{ where } q = 0, 1, 2 \ldots$$

which is the condition that the transverse rotation frequency of particle within the beam is an integer multiple of the "focusing frequency," $m\,\Omega_{zo}$ — a condition which allows resonant transfer of energy from the longitudinal to transverse degrees of freedom and, consequently, exponential growth of the betatron oscillation amplitude.

As B_z is increased during acceleration, one typically would not wish to increase B_θ simultaneously since this would require significant additional energy storage. The result is that the operating point of the accelerator will move from right to left in Fig. 1. Consequently, the accelerator should be run in the left-most stable band to avoid crossing unstable bands. These considerations require $m > b$ at injection and force the use of a large number of field periods in the design of the strong-focusing system. The left-most unstable band, corresponding to $q = 0$, is due to the beam space-charge and rapidly disappears during acceleration since the self-field index, n_s, is proportional to γ_o^{-3}, where γ_o is the relativistic factor. The left-most stable band, therefore, becomes broader during acceleration; the first stable band is at its most narrow at injection, when γ_o is smallest.

We next consider the important question of containment of particles whose average momentum is not matched to the vertical betatron field, i.e., the question of the momentum compaction of this configuration. In order to address this question we have examined numerically the behavior of single particle orbits, neglecting beam self fields but employing the full Bessel function representation of the $l = 0$ focusing field. Figure 2 shows the allowed mismatch, $\delta P/P_o$, plotted against $\epsilon \equiv \delta B_\theta/B_\theta$ for $B_{\theta o} = 2kG$, $B_{zo} = 118G$, $n = \frac{1}{2}$, $r_o = 100$ cm and $m = 30$. This plot is generated numerically by launching particles on the minor axis along the toroidal direction with various amounts of mismatch. The figure shows the largest mismatch for which the calculated orbits are contained in a 10 cm minor radius chamber. Containment of particles with a mismatch of $\pm$ 20% is obtained for $\epsilon \simeq 0.2$. We stress that the momentum compaction of this configuration is due to the alternating gradient field of the "bumps," though the phase shift per period is dominated by the average value of the toroidal field. Using Eq. (2), with $n_s = o$, a perturbative calculation valid for small values of ϵ, of the momentum compaction factor, gives

$$\frac{\delta r/r_o}{\delta P/P_o} \approx 2\left[1 - \left(\frac{\epsilon m b}{2}\right)^2 \frac{1}{m^2 - b^2}\right] \tag{3}$$

which holds only for $m > b$. One sees in Eq. (3) the helpful effect of a bumpy torus field.

In conventional betatrons, resonances are automatically avoided by increasing the particle momentum and the vertical magnetic field in synchronism. The introduction of non-synchronous fields (a fixed toroidal field, for example) makes the betatron wavelengths energy dependent, which can lead to the crossing of resonances driven by field errors during acceleration. As in all strong-focusing devices, the occurrence of orbital resonances plays an important role in the operation of the bumpy-torus betatron. Using the Floquet solutions to Eq. (2) it is possible to obtain a condition for the integer resonances, when space-charge effects may be neglected:

$$\psi_1(\pi) = \cos\left[\pi\left(\frac{b + 2k}{m}\right)\right] \tag{4}$$

where $\psi_1(\theta_m)$ is the solution to Eq. (2) with $\delta P \equiv C$ satisfying $\psi_1(0) \equiv 1$, $\psi_1'(0) = 0$ and where k is an

integer, the Fourier component number of the dipole field error. Equation (4) provides the basis for numerical calculation of contours in the stability plane on which Eq. (4) is satisfied for a given k; an example is given in Fig. 3.

If all the fields cannot be made synchronous with the particle energy, the effect of resonant instabilities might be minimized by making the energy gain per pass large. Other possibilities for coping with resonance crossings are currently under investigation.

III. SUMMARY

In conclusion, we find the spatially alternating transverse magnetic field gradient associated with a bumpy-torus leads to a potentially interesting strongly-focused accelerator configuration which is seen to have a region of stable orbits, and to have a significant bandwidth in allowed mismatch between the vertical magnetic field and the particle momentum.

ACKNOWLEDGMENT

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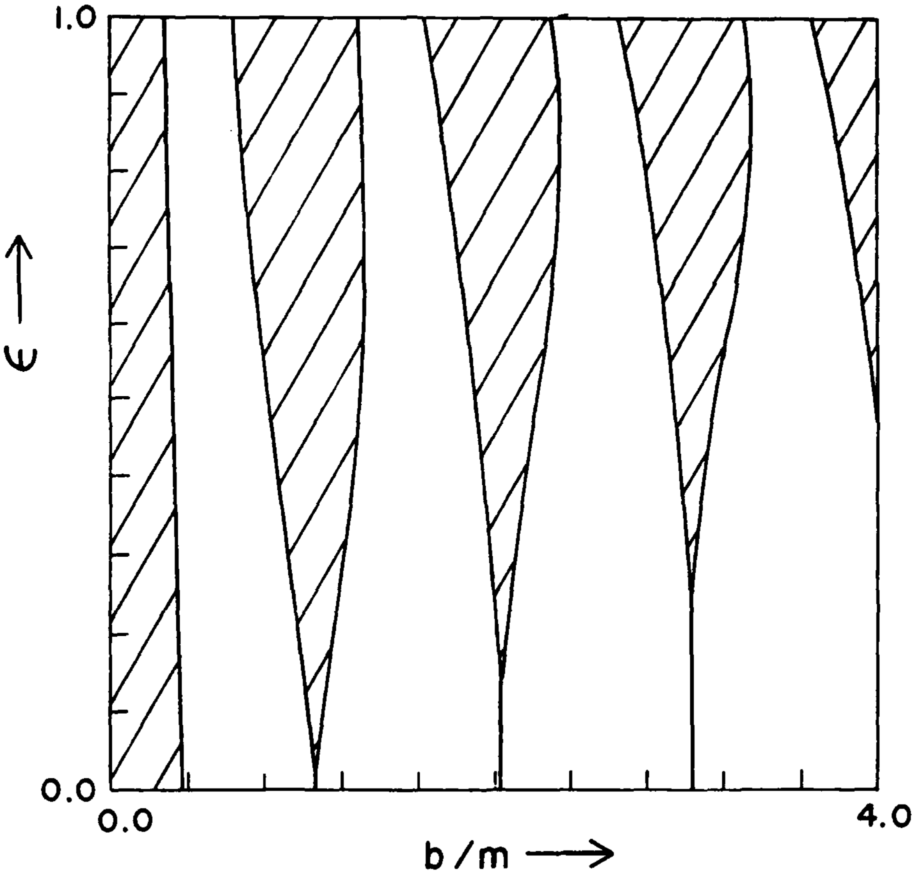

Fig. 1 — Stability plane for bumpy-torus betatron, for the case $n_s = m = 30$. The shaded regions are unstable for particle motion.

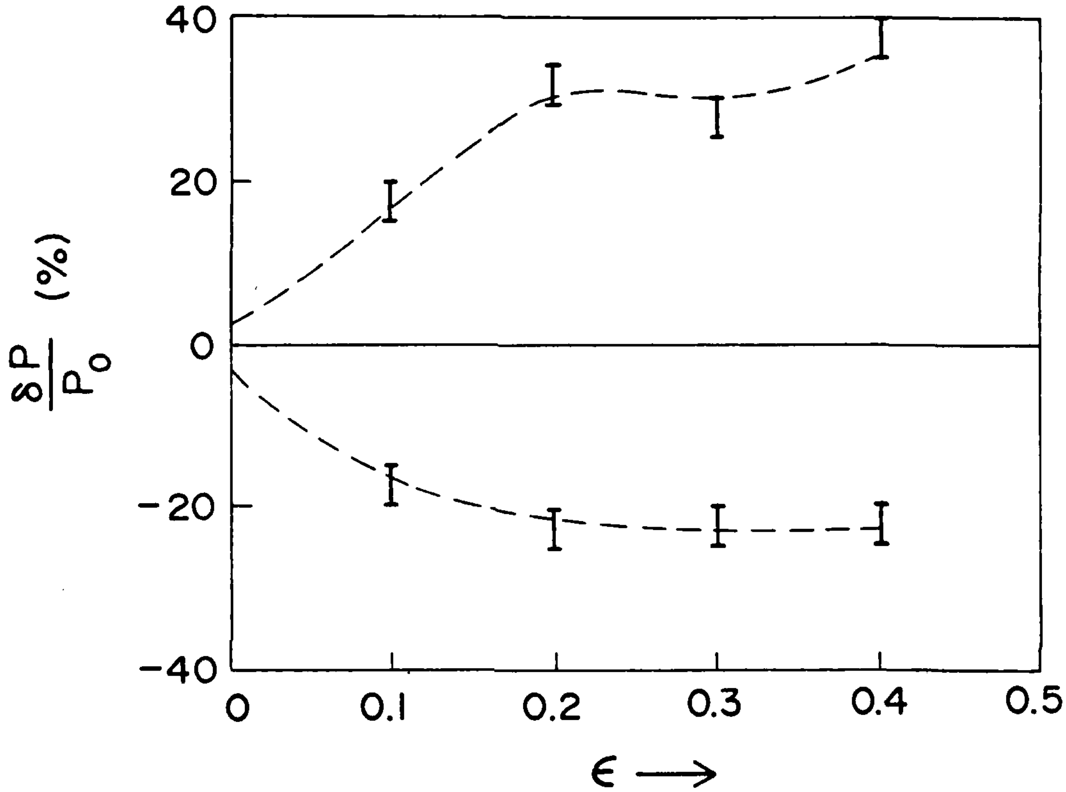

Fig. 2 — Single particle bandwidth. Data points indicate the maximum value of momentum mismatch tolerated by the device vs the bump size, ϵ, for particles initialized on the minor axis, for the specific case $B_{z0} = 118G$, $B_{\theta 0} = 2kG$, $r_0 = 100$ cm, $m = 30$

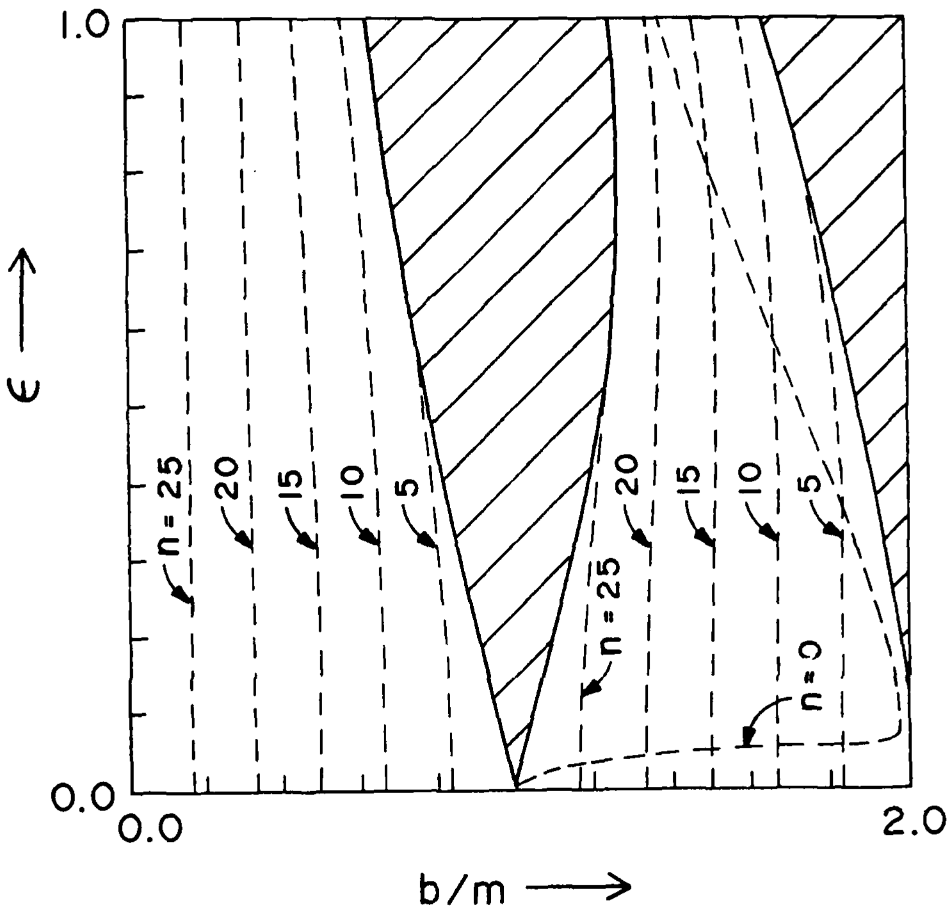

Fig. 3 — Stability plane for bumpy-torus betatron, with the single particle resonance lines $n = 0, 5, 10, 15, 20, 25$, indicates for the case $n_s = 0$, $m = 30$

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